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PAPER

Viscous drag friction acting on a fluid drop confined in between two plates

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Dealing with a small amount of liquid has become increasingly important in recent applications in many fields such as biology, chemistry and medicine. In such a context, viscous drag friction acting on fluid drops in confined geometries is an indispensable fundamental issue, as the Stokes' friction law for a sphere in the bulk is useful in many physical processes. We study here, in a quasi two-dimensional confined space (*i.e.*, in a Hele-Shaw cell), such viscous drag friction opposing gravitational drive. As a result, we establish in a clear way scaling laws for the viscous drag friction in different regimes. These scaling laws replace, in the confined geometry, the well-known Stokes' friction law. The proposed laws are unexpectedly simple in spite of the potential subtle effects of liquid thin films existing between the drop and the cell plates, thanks to the principle of minimal dissipation in viscous hydrodynamics.
PACS numbers: 47.55.D- Drops and bubbles; 89.75.Da Systems obeying scaling laws; 68.15.+e Liquid thin films; 83.50.Lh Slip boundary effects; 47.57.Bc Foams and emulsions.

Introduction

The dynamics of bubbles and drops, familiar in daily life, are important not only in the physical sciences[1–6] but also in a variety of practical situations such as emulsification, formation of spray and foams,[7–9] commercial ink-jet printing,[10] and lab-on-a-chip manipulations.[11] An important issue is the determination of scaling laws representing the essential physics[12] as in many cases, such as the lifetime of a bubble in a viscous liquid[13] and the contact dynamics of a drop to another drop[14,15] or to a solid plate.[16] Here, we report on clear scaling laws experimentally obtained for viscous friction acting on a fluid drop in a confined space. These friction laws replace the well-known Stokes' law in situations which have become important recently, *e.g.*, in microfluidics applications. For this purpose we study, in a quasi two-dimensional bath of oil in a Hele-Shaw cell, the dynamics of a rising bubble and of a setting (sinking) aqueous drop due to gravity.

The rising bubble in a Hele-Shaw cell is theoretically discussed by Taylor and Saffman in a pioneering paper[17] in 1958 (earlier than Bretherton's paper[18]. Related bubble dynamics in tubes is discussed in ref. 19.). They derived a family of exact solution for a bubble moving in the Hele-Shaw cell but could not select a unique solution from it in a rigorous way so that they made an *ad hoc* speculation. This degeneracy of solution was discussed by a perturbation theory and a numerical study by Tanveer,[20] which support the speculation. In addition, there are many theoretical works on fluid drops in the Hele-Shaw cell geometry (*e.g.*, ref. 21–24). Different from these theoretical attempts, here, as we see below, we should pay a special attention to the existence of a thin liquid film surrounding the bubble.

As for experimental studies, a number of researches on bubble-rising in a Hele-Shaw cell have been performed[25–27] (see, *e.g.* ref. 28 and 29 for related bubble dynamics in different contexts). However, systematic and quantitative studies in a constant velocity regime are all concerned with the case where the cell is strongly inclined nearly to a horizontal position and several discrepancies with the study in ref. 17 have been reported. Here, we study an "opposite" case of vertical cell. As a result, our theory and experiment agree well with each other as seen below: we establish the counterpart of the Stokes' friction law in a Hele-Shaw cell geometry for a rising bubble, together with such counterparts for a setting drop in different regimes. Although our problem includes potentially intriguing liquid thin films between the drop and the cell plates, the emergent laws are simple enough to be useful in practical applications. This is due to the principle of minimum viscous dissipation in hydrodynamics.

Experimental

Our experiments were performed in a Hele-Shaw cell made of transparent acrylic plates positioned in parallel at a millimetre distance D by spacers.[30–32] We conducted three series of experiments. In the first series, we fill polydimethylesiloxane (PDMS) in a vertically positioned Hele-Shaw cell (the cell thickness direction is horizontal) and inject an air bubble of centimetre size at the bottom which rises in the cell (Fig. 1a). In the second and third series, we instead inject a glycerol drop at the top, which sets (or sinks) in PDMS contained in a Hele-Shaw cell. In the second series (Fig. 1b) the drop is more viscous than the surrounding oil while in the third series (Fig. 1c) the drop is less viscous.

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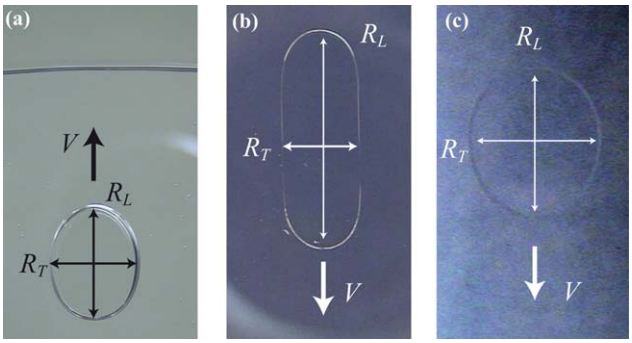

Fig. 1 Front view of a rising bubble in PDMS filled in a Hele-Shaw cell (a). Setting drop of glycerol in the PDMS where the drop is more viscous than the surrounding oil (b) and where the drop is less viscous (c). The fluid drops are not circular and characterized by transverse and longitudinal sizes R_T and R_L as in the figure.

Note that since PDMS completely wets the plates the contact line between the drop (bubble or glycerol) and the plate is absent and a thin liquid film of PDMS exists between the fluid drop and the plate. The cell is much larger than the fluid drop to avoid unwanted effects on the flow from the side- and bottom-spacers of the cell.

Results and discussion

Rising bubble

Let us first examine the case of a rising bubble. The rising speed is constant as in Fig. 2a for a given set of parameters: cell-thickness D, kinematic viscosity ν, and transverse and longitudinal sizes R_T and R_L. The dependences of the constant velocity on D and R_T are shown in Fig. 2b and c. The bubble is slightly elongated as in Fig. 1a and in Fig. 2d. The experimental parameters are given in Tab. I.

As a prototype of the ensuing discussion, consider first a trivial case of a spherical bubble of radius R rising with velocity V in a three-dimensional liquid bath much larger than the bubble (Fig. 3a). The bubble makes a flow near the bubble in a region of volume around R^3 with a velocity gradient of the order of V/R. Since the dissipation per unit volume per unit time scales as viscosity η times the squared velocity gradient,[33] the total dissipation scales as $\eta(V/R)^2R^3$. Balancing this energy with the gain in the gravitational energy per unit time, $\rho g R^3 V$, we obtain a scaling relation $V \simeq \rho g R^2/\eta$, as is well-known. Here, ρ and g are the density of the liquid and the gravitational constant, respectively.

Consider next the case in which a bubble is squashed in a Hele-Shaw cell. A thin film between the plate and the bubble as illustrated in the side view in Fig. 3b seems important for dissipation. However, air viscosity is so low that it could not drag the thin liquid film of thickness h efficiently and the strong dissipation associated with velocity gradient V/h could be avoided: bubble could slip over the thin film. This assumption is justified shortly by our experiment in a clear way and is consistent with the Bretherton's argument as discussed below. Under this assumption, the important velocity gradient for dissipation should not be V/R but V/D because $D \ll R$. Thus, we balance the total dissipation per unit time $\eta(V/D)^2R^2D$ with the gravitational energy gain per unit time $\rho g R^2 D V$ to obtain a scaling relation

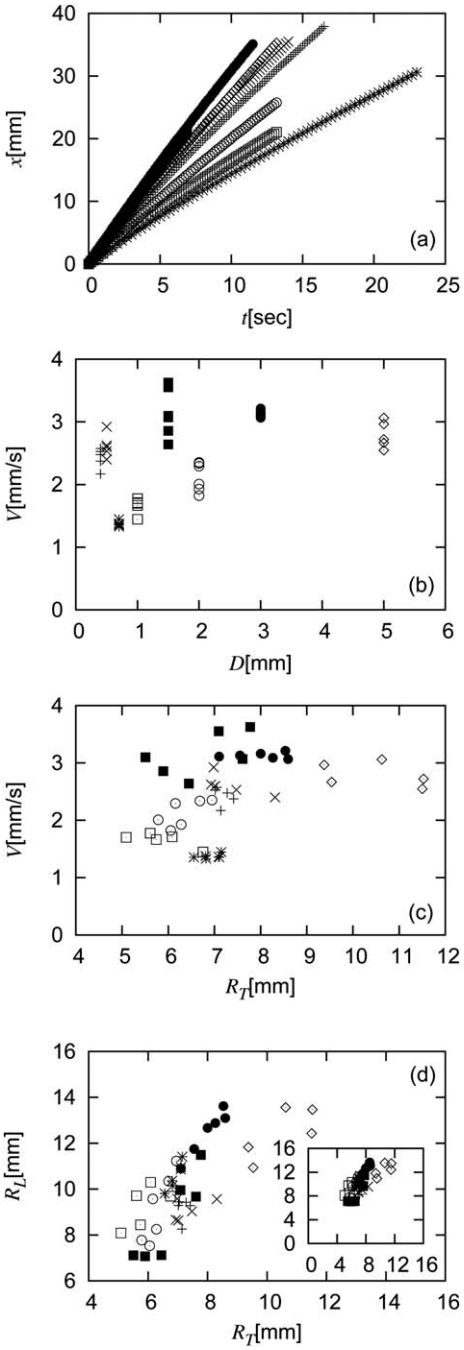

Fig. 2 (a) Central position x of the bubble as a function of time t. (b) Rising velocity V as a function of the cell thickness D. (c) V as a function of the transverse size R_T. (d) Shape of bubbles: the longitudinal size R_L is plotted as a function of R_T. Symbols are specified in Table 1.

Table 1 Cell thickness D and kinematic viscosity ν of PDMS in the rising bubble experiments. The density of PDMS is about 1 g cm^{-3} (0.965, 0.970 and 0.975 g cm^{-3} at ν = 100, 500–3000 and 5000–10000 cS, respectively)

	+	×	✳	□	■	○	●	◇
D/mm	0.4	0.5	0.7	1.0	1.5	2.0	3.0	5.0
ν/cS	100	100	500	1000	1000	3000	5000	10000

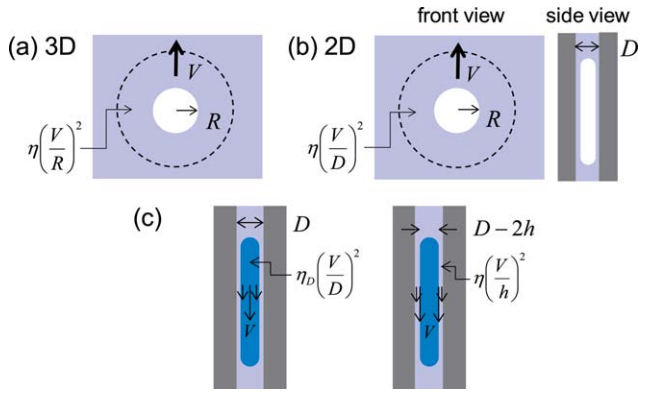

Fig. 3 (a) A spherical bubble rising in a liquid bath where the dissipation $\eta(V/R)^2$ localized in the volume R^3 is dominant. (b) A squashed bubble rising in a Hele-Shaw cell where the dissipation $\eta(V/D)^2$ localized in the volume R^2D is dominant. (c) Side view of a glycerol drop going down in PDMS contained in the Hele-Shaw cell: (right) solid-like thin film of PDMS (thickness h) resulting in the Poiseuille flow inside the drop *versus* (left) air-like thin film resulting in the plug flow inside the drop.

$$V \simeq \rho g D^2/\eta \tag{1}$$

where the previous scale R is replaced with a new scale D.

Bretherton employed a similar assumption for a bubble rising in a tube,[18] where the dominant friction force $\eta V/h^2$ comes from the dynamic meniscus whose scale is severely restricted to l. Outside this dynamic meniscus no significant dissipation occurs as we assumed in the above: the global dissipation in the thin film is avoided. At the meniscus, we balance the local dissipative force (per unit volume) $\eta V/h^2$ with the gradient of the capillary pressure $\gamma/(lD)$ and match the curvatures from the both sides ($1/D \simeq h/l^2$), where γ is the surface tension. Thus, we obtain the Bretherton's law $h \simeq D(\eta V/\gamma)^{2/3}$. The dissipation $\eta(V/h)^2lRh$ associated with the force $\eta V/h^2$ is less dominant than the above dissipation $\eta(V/D)^2R^2D$ in our case because the volume ($\simeq lRh$) associated with this dissipation is much smaller than that ($\simeq R^2D$) with the above dissipation.

Consider finally an extreme case where R_L is much larger than R_T to include the asymmetry factor R_L/R_T into our theory. In this limiting case, the bubble should completely slip over the thin film around the central region of the bubble; the dominant dissipation occurs only in the head and tail regions (of volume R_T^2D) of a bubble, which is similar to the Bretherton's situation. In other words, the volume of dissipating region changes from R^2D to R_T^2D, while the gravitational energy gain per unit time is trivially replaced with $\rho g R_T R_L D V$. In this way, we obtain a scaling relation $V \simeq (R_L/R_T)\rho g D^2/\eta$, which is expressed in a dimensionless form

$$(R_T/R_L)\mathrm{Ca} \simeq (\kappa D)^2 \tag{2}$$

Here, $\mathrm{Ca} \equiv \eta V/\gamma$ is the capillary number and $\kappa^{-1} = \sqrt{\gamma/\rho g}$ is the capillary length. We replot Fig. 2b with rescaling according to eqn (2) in Fig. 4; all the scattered data points (more than 40) in Fig. 2b almost completely collapse onto the predicted line. In addition, we have confirmed explicitly that the above scaling law can be derived from a general but non-unique result given in ref. 17 as a special limiting case.

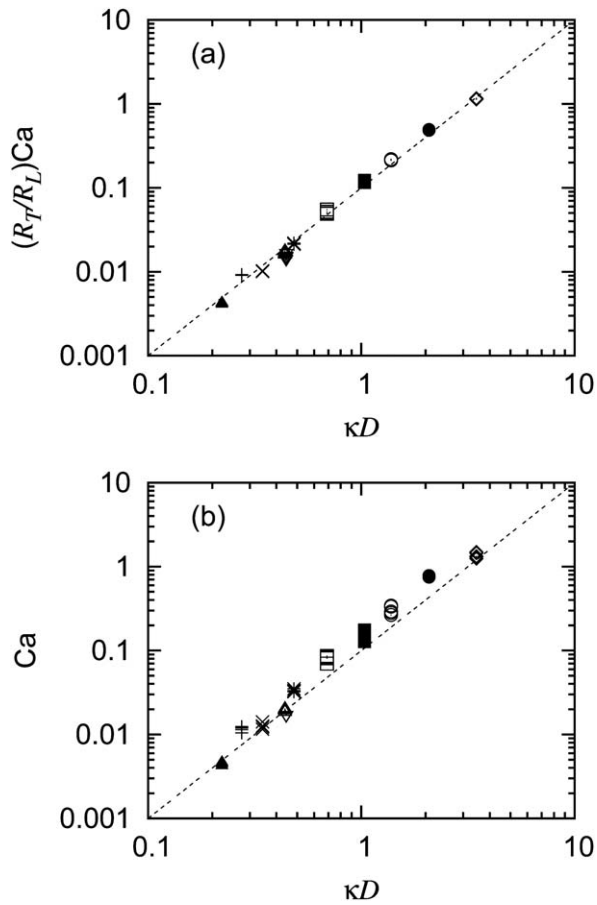

Fig. 4 Rescaled log plot of Fig. 2b: Ca and κD are the renormalized velocity and cell thickness, respectively. All the data points (more than 40) in Fig. 2b–c almost exactly collapse on to the theoretical line with slope 2 indicated by a straight dashed line. The bottom plot shows the same plot but without the asymmetry factor R_T/R_L. Omission of this factor slightly disgraces the collapse, since the factor is not equal (but close) to one. These plots actually include the data obtained in the experiment of a sinking glycerol drop in PDMS which surrounds the drop and is more viscous than the drop (see below). Symbols are specified in Table 1 and 3.

Setting drops

Now we examine the case of setting (sinking) of a heavier glycerol drop in lighter PDMS; we dilute glycerol with water to change the viscosity of the drops.

The case of viscous drops. We first consider the case where the drop viscosity is larger than the surrounding oil. The sinking velocity is constant for a given parameter set and the constant velocity depends on D, ν, R_T and R_L as before (Fig. 5a). The shape is again asymmetric but the degree is stronger than in the previous case (Fig. 5b and 1b).

We intentionally used a PDMS (outside liquid) of lower viscosity to demonstrate a novel feature in Fig. 5. The Stokes friction $6\pi\eta RV$ acting on a sphere or drop of radius R in a flow of velocity V is dependent not on the viscosity of drop η_D but on the outside viscosity η if η_D is much larger than η.[34] However, as seen below, the opposite situation where friction depends not on η but on η_D is expected when a drop is squashed in a Hele-Shaw cell.

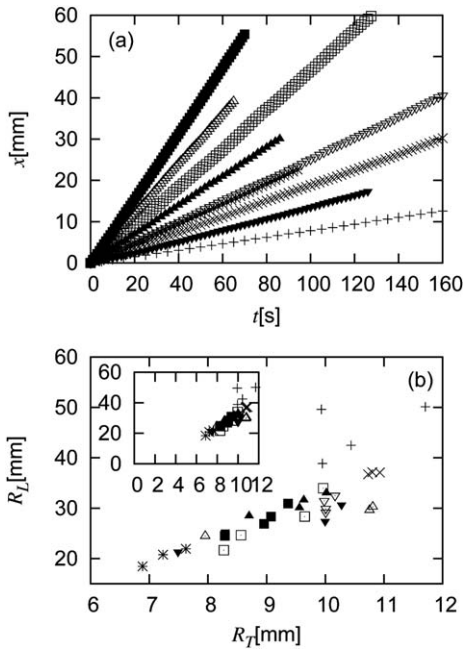

Fig. 5 (a) Central position x of the glycerol drop as a function of time t. (b) Shape of glycerol drops: the longitudinal size R_L is plotted as a function of R_T. Symbols are specified in Table 2.

Table 2 Cell thickness D and kinematic viscosity of glycerol drops ν_D in the experiments of a glycerol drop sinking in a less viscous PDMS. The density ρ_D of glycerol aqueous solution is 1.25 g cm^{-3} with an exception: 1.26 for white triangle △. Viscocity of PDMS is negligibly small (viscosity and density of PDMS are 1 cS and 0.818 g cm^{-3} with an exception: they are 10 and 0.935 for black inverse triangle ▼)

	△	▲	✳	▽	▼	+	×	□	■
D/mm	1.0	1.0	1.0	1.0	1.0	0.5	0.7	1.2	1.5
ν_D/cS	316	585	623	966	656	683	684	712	718

To understand this point, we go back to the illustration in Fig. 3a, but with the direction of V reversed (because the density of the glycerol is larger than that of PDMS) and with the viscosity and density of the sphere (or drop) being significantly higher. The system may avoid making any flow inside the sphere to avoid unnecessary strong dissipation just as in the previous case of the thin film of PDMS. Then the total dissipation per unit time is again given by $\eta(V/R)^2R^3$ with only the outside fluid viscosity appearing. The gravitational energy gain per unit time ρgR^3V is simply replaced with $\Delta\rho gR^3V$ where $\Delta\rho$ is the difference of density of the two fluids. Balancing these two energies, we obtain a relation $V \simeq \Delta\rho gR^2/\eta$, which is independent of the viscosity inside of the drop, as in the Stokes law.

However, when this highly viscous drop is squashed in the Hele-Shaw cell, the situation changes significantly. As before, the thin PDMS film existing between the drop and cell plate tends to avoid strong dissipation associated with a velocity gradient V/h; namely, this thin film acts like a solid sheet to help produce a velocity gradient V/D inside the drop; whether the thin film tends to act like a solid sheet developing a Poiseuille flow inside the drop or like an air sheet developing a plug flow inside the drop (see Fig. 3c) can be judged by comparing the viscous dissipation per unit time per unit area of the film: the dissipation in the former case of solid scales as $\eta_D(V/D)^2D$ and that in the latter case of air as $\eta(V/h)^2h$, with η_D being the viscosity of the drop, and these two estimates give a criterion for the solid-like regime to become less dissipative:

$$h \ll (\eta/\eta_D)D. \tag{3}$$

We assume that this condition is satisfied, which is justified shortly by our experimental data. This assumption of solid-like behavior is also supported by the Bretherton's law mentioned above: by employing this law, this assumption becomes equal to $Ca^{2/3} \ll \eta/\eta_D$ which is well satisfied in the parameter range of our experiment. Note that the local dissipation inside the dynamic meniscus is again negligible compared with more global dissipation associated with the velocity gradient V/D developed inside the drop.

Under this assumption of solid-like behavior, the total dissipation per time is counted as $\eta_D(V/D)^2R_TR_LD$, which is balanced by the gravitational gain per time $\Delta\rho gR_TR_LDV$. Here, we should stress the volume of dissipating region is not R_T^2 but R_TR_L in this case even if $R_T \gg R_L$; this is because dissipation occurs inside the drop. In this way we obtain a scaling relation:

$$V \simeq \Delta\rho gD^2/\eta_D \tag{4}$$

As expected, now the velocity is governed by the inside viscosity η_D, which is the novel feature announced above. In addition, we have confirmed that the scaling law given in eqn (4) can be derived by taking an appropriate limit given in the formal result in ref. 17.

To check that the viscosity inside the drop truly determine the dynamics, we first compare velocities for a fixed cell thickness. Although R_T and R_L are not fixed for the data, if the prediction in eqn (4) is correct, we expect that V is inversely proportional to a rescaled drop viscosity $\eta_D/\Delta\rho$, which is confirmed in Fig. 6a. To further confirm the prediction, we plot V as a function of $\Delta\rho gD^2/\eta_D$ by performing experiments for various D; if eqn (4) is correct all the data should collapsed onto a straight line, which is reasonably well confirmed in Fig. 6b. The reason we cannot get a clear collapse of the level of the bubble case as in Fig. 4 is as follows. In the present case of high viscous drop, there should be a dissipation of the order of $\eta_D(V/R_T)^2$ inside the drop. However, in the current experiment, it is difficult to well separate the scale D from that of R_T because it is not feasible to make the drop size larger in a controlled way. Indeed, if we limit the data in which this separation is well ($D < R_T/10$), we find a better collapse as indicated in the inset of Fig. 6b (the data in Fig. 6a are also selected to satisfy this condition). In this way, we conclude that the thin but less viscous PDMS film surrounding the more viscous drop does not contribute the dynamics, which suggests the condition in eqn (3) is satisfied.

The case of less viscous drops. When the drop viscosity is smaller than the surrounding oil, drops again sink in the surrounding oil at constant velocity. However, the shape of the drop becomes less asymmetric as in Fig. 1c and similar to rising

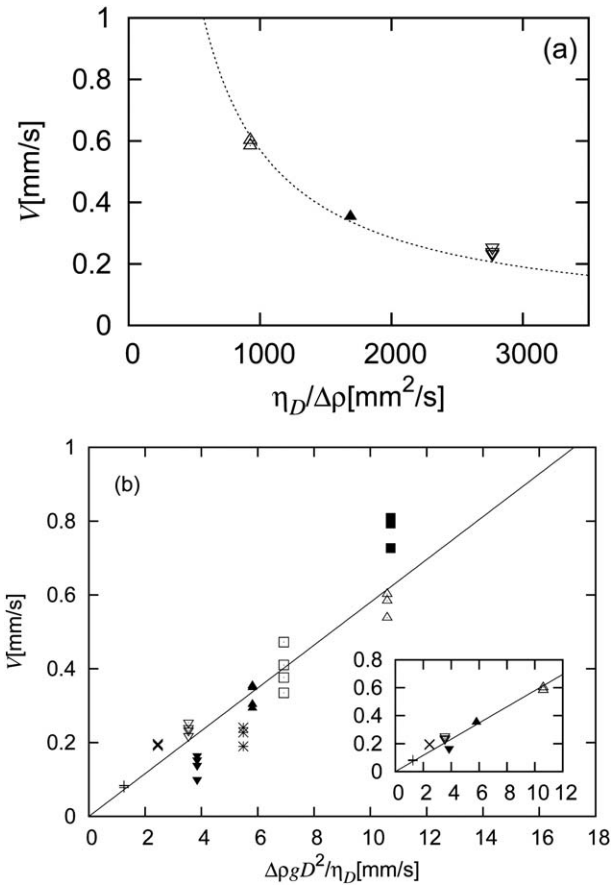

Fig. 6 (a) Velocity V as a function of a renormalized drop viscosity $\eta_D/\Delta\rho$, which confirms that high viscosity of the drop determines the dynamics. (b) V as a function of the squared cell thickness D^2. The inset is the same plot but only of data with well separation between D and R_T for which the approximation becomes better. Symbols are specified in Table 2.

bubbles. Indeed, when we rescale the plot according to eqn (2) the data collapse onto the data obtained from the rising bubbles. This is shown in Fig. 4 as indicated in the caption. On the plots, the data represented by ▲, ▽ and △ (see Table 3) are obtained from the experiment of a glycerol drop sinking in a more viscous PDMS. These marks are plotted at $\kappa D \simeq 0.2$, 0.45 and 0.45, respectively, and are recognized easier in Fig. 4b because they are collapsed too well in Fig. 4a. As understood from this good collapse of the data, the dynamics of a less viscous drop setting in a viscous surrounding oil is physically the same with that of rising bubbles. Only the direction of the movement is different due to the relation between relative densities: drops are heavier while bubbles are lighter than the surrounding fluid.

Table 3 Cell thickness D and kinematic viscosity of PDMS ν in the experiment of a glycerol drop sinking in a more viscous PDMS. Viscosity of glycerol aqueous solution is negligibly small ($\nu_D = 15.2$ cS and $\rho_D = 1.17$ g cm^{-3})

	▲	▽	△
D/mm	1.0	2.0	2.0
ν/cS	100	100	500

Conclusion

We have shown that important are the dissipation associated with a velocity gradient V/D over the cell thickness D in all the cases studied here. However, the region where this gradient develops is different depending on the relative importance of the fluid drop viscosity to that of the surrounding fluid (here, a fluid drop includes an air bubble). When the surrounding oil is more viscous this gradient develops in the surrounding liquid around the head and tails of the drop over a volume about $R_T^2 D$. When the drop is more viscous, however, the gradient develops inside the whole drop over a volume about $R_T R_L D$. These scaling views reveal that the Stokes friction law scaling as ηVR in the bulk is replaced in a Hele-Shaw cell by $\eta VR_T^2/D$ for a viscous surrounding fluid (regardless of whether the fluid drop is rising or setting) and $\eta_D VR_T R_L/D$ for a viscous fluid drop:

$$\eta VR \Rightarrow \begin{cases} \eta VR_T^2/D & \text{viscous surrounding fluid} \\ \eta_D VR_T R_L/D & \text{viscous fluid drop} \end{cases} \quad (5)$$

In the latter case, a high viscosity of drop determines the dynamics, which is unexpected from the well-known Stokes friction in the bulk. This unexpected feature originates from the existence of thin film between the drop and cell plates, inside which dissipation is avoided. The principle we confirmed in this paper that strong viscous dissipation tends to be avoided (inside a thin liquid film or inside a viscous drop), together with the proposed simple friction laws for confined fluid drops should play significant roles in understanding a variety of viscous dynamics in both academic and applied problems.

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References

1 D. Richard, C. Clanet and D. Quéré, *Nature*, 2002, **417**, 811.
2 P. Doshi, I. Cohen, W. W. Zhang, M. Siegel, P. Howell, O. A. Basaran and S. R. Nagel, *Science*, 2003, **302**, 1185.
3 Y. Couder, S. Protiere, E. Fort and A. Boudaoud, *Nature*, 2005, **437**, 208.
4 W. D. Ristenpart, J. C. Bird, A. Belmonte, F. Dollar and H. A. Stone, *Nature*, 2009, **461**, 377.
5 E. Kelley and M. Wu, *Phys. Rev. Lett.*, 1997, **79**, 1265.
6 J. Bird, R. De Ruiter, L. Courbin and H. Stone, *Nature*, 2010, **465**, 759.
7 A. Frohn and N. Roth, *Dynamics of Droplets* (Springer, Berlin, 2000).
8 D. Weaire and S. Hutzler, *The Physics of Foams* (Clarendon Press, Oxford, 1999).
9 I. Cantat, S. Cohen-Addad, F. Elias, F. Graner, R. Höhler, O. Pitois, F. Rouyer, and A. Saint-Jalmes, *Les mousses: structure et dynamique* (Belin, Paris, 2010).
10 P. Calvert, *Chem. Mater.*, 2001, **13**, 3299.
11 T. M. Squires and S. R. Quake, *Rev. Mod. Phys.*, 2005, **77**, 977.
12 P.-G. de Gennes, F. Brochard-Wyart, and D. Quéré, *Gouttes, Bulles, Perles et Ondes*, 2nd edn (Belin, Paris, 2005).
13 G. Debrégeas, P.-G. de Gennes and F. Brochard-Wyart, *Science*, 1998, **279**, 1704.
14 D. G. A. L. Aarts, H. N. W. Lekkerkerker, H. Guo, G. H. Wegdam and D. Bonn, *Phys. Rev. Lett.*, 2005, **95**, 164503.
15 M. Yokota and K. Okumura, *Proc. Nat. Acad. Sci. (USA)*, 2011, **108**, 6395.

16 J. C. Bird, S. Mandre and H. A. Stone, *Phys. Rev. Lett.*, 2008, **100**, 234501.
17 G. Taylor and P. G. Saffman, *Q. J. Mech. Appl. Math.*, 1959, **12**, 265.
18 F. P. Bretherton, *J. Fluid Mech.*, 1961, **10**, 166.
19 C. Clanet, P. Héraud and G. Searby, *J. Fluid Mech.*, 2004, **519**, 359.
20 S. Tanveer, *Phys. Fluids*, 1986, **29**, 3537.
21 J. Eggers, *Rev. Mod. Phys.*, 1997, **69**, 865.
22 P. Constantin, T. F. Dupont, R. E. Goldstein, L. P. Kadanoff, M. J. Shelley and S.-M. Zhou, *Phys. Rev. E*, 1993, **47**, 4169.
23 R. E. Goldstein, A. I. Pesci and M. J. Shelley, *Phys. Rev. Lett.*, 1995, **75**, 3665.
24 P. D. Howell, *J. Eng. Math.*, 1999, **35**, 251.
25 T. Maxworthy, *J. Fluid Mech.*, 1986, **173**, 95.
26 A. R. Kopf-Sill and G. M. Homsy, *Phys. Fluids*, 1988, **31**, 18.
27 S. R. K. Maruvada and C.-W. Park, *Phys. Fluids*, 1996, **8**, 3229.
28 C. Ybert and J.-M. Di Meglio, *Eur. Phys. J. E: Soft Matter Biol. Phys.*, 2000, **3**, 143.
29 J. Magnaudet and I. Eames, *Annu. Rev. Fluid Mech.*, 2000, **32**, 659.
30 A. Eri and K. Okumura, *Phys. Rev. E*, 2007, **76**, 060601(R).
31 A. Eri and K. Okumura, *Phys. Rev. E*, 2010, **82**, 030601(R).
32 Y. Takehara, S. Fujimoto and K. Okumura, *Europhys. Lett.*, 2010, **92**, 44003.
33 L. Landau and E. Lifshitz, *Fluid Mechanics*, 2nd edn (Pergamon, 1987).
34 H. Lamb, *Hydrodynamics*, 6th edn (Dover, 1993).