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Discontinuity in the In-plane to Out-of-plane Transition of Kirigami

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Recently, we have shown that kirigami's high extensibility emerges as a result of the transition in its deformation modes and that the transition observed through the force-elongation curve can be discontinuous. Using our previous model, we recover the discontinuous transition, deriving an expression that characterizes the force discontinuity at the point of transition. We further show that this expression well accounts for our experimental results.

"Kirigami" is originally a Japanese art technique using paper and scissors ("Kiri" means cutting, "gami" means paper in Japanese), which can be regarded as a variation of "Origami" in scientific fields. These techniques can impart mechanical and functional properties to sheet materials using macroscopic patterning.[1] The basic kirigami structure with parallel slits makes sheet materials highly stretchable, leading to a nonlinear mechanical response in the tensile test (see Fig. 1).[2–6] This response is accompanied by a quasi-plateau region that indicates high stretchability. Our previous study[2] showed experimentally that the force drops off after the transition, as shown in Fig. 1(b), which suggests that the transition is discontinuous.

In the present study, we extend our previous model to recover the discontinuity in the transition, deriving an expression for quantifying the force drop at the transition. We also present our experimental results and demonstrate the agreement between theory and experiment.

In our previous work,[2] we showed a nonlinear mechanical response using a simplified one-column structure of kirigami as shown in Fig. 1, where the force F is plotted as a function of the elongation Δ. We focused on the initial linear regime [Fig. 1(b)]. The maximum point in the regime is referred as the transition point, at which the transformation occurs from in-plane to out-of-plane deformation [Fig. 1(c)]. We constructed a simple kirigami model by estimating the deformation energy of the structure as a series of slightly bent beams. The beam element can undergo either in-plane or out-of-plane deformation for a given elongation $\delta = \Delta/(2N)$, where $2N$ is the number of units. One of the two deformations with smaller energy is expected to be realized in practice. The two deformation energies of the element were calculated as $U_{\rm in} = kE(hd^3/w^3)\delta^2$ and $U_{\rm out} = kE(dh^3/w^3)(\delta^2 + 2d\delta)$, respectively. Here, d and w denote the distance between slits and the length of slits, respectively [Fig. 1(a)], while k is a numerical coefficient. We discuss these energies in normalized forms using the energy scale kEd^2h^5/w^3 and the length scale d:

$$\tilde{U}_{\rm in}(\tilde{\delta}) = \tilde{h}^{-2}\tilde{\delta}^2 \qquad (1)$$

$$\tilde{U}_{\rm out}(\tilde{\delta}) = \tilde{\delta}^2 + 2\tilde{\delta} \qquad (2)$$

For small $\tilde{\delta}$ $(= \delta/d)$, the in-plane and out-of-plane energies are proportional to $\tilde{\delta}^2$ and $\tilde{\delta}$, respectively.

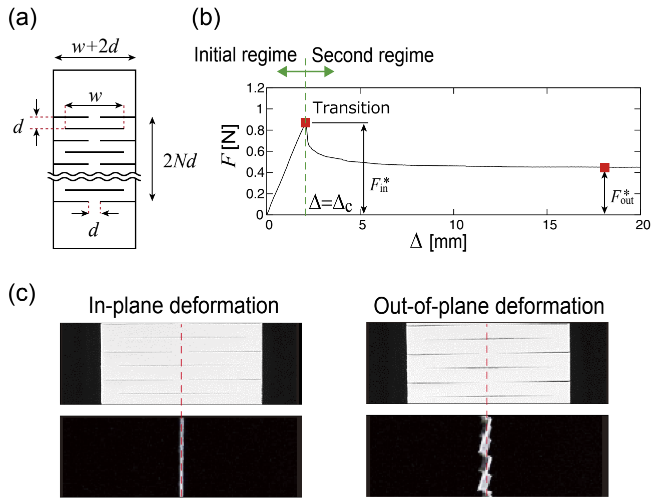

Fig. 1. (Color online) (a) Sample geometry. (b) The initial and second regimes of the force-elongation curve obtained for the case in which $(h, d, w) = (0.22, 2, 17)$ (in the unit of mm). (c) Snapshots of kirigami sample during deformation viewed from the front and side. Left and right photographs were taken before and after transition, respectively.

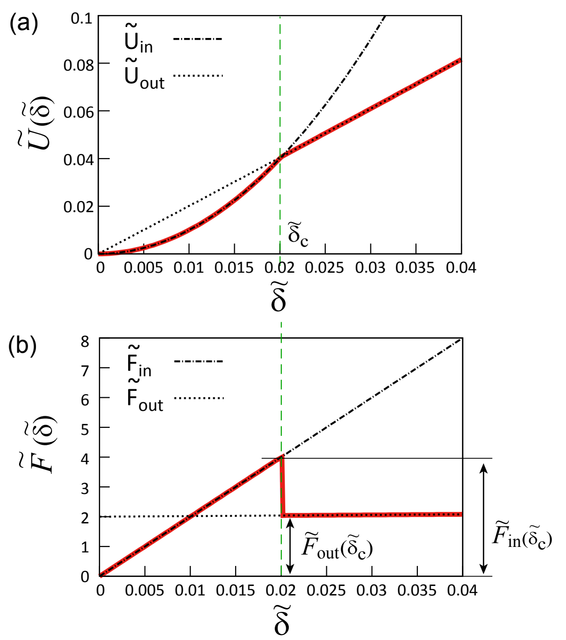

Fig. 2. (Color online) (a) Normalized in-plane and out-of-plane energies plotted as a function of normalized elongation $\tilde{\delta}$ [Eqs. (1) and (2)]. (b) Normalized in-plane and out-of-plane forces plotted as a function of normalized elongation $\tilde{\delta}$. The (red) bold curves are the predictions of the energy and force curves, respectively, in the present theory.

For completeness, analytical calculations were performed without assuming that $\tilde{\delta}$ and $\tilde{h}$ are small. For this purpose, we kept the term $\tilde{\delta}^2$ in Eq. (2), which is negligible for small $\tilde{\delta}$. [Note that $\tilde{h}$ $(= h/d)$ is generally small in kirigami geometry, as seen below in our experiments.]

Equations (1) and (2) are plotted in Fig. 2(a) for $\tilde{h} = 0.1$. The bold curve corresponds to the lower energy among the two energy curves, which are interchanged at $\tilde{\delta} = \tilde{\delta}_c$, and thus corresponds to the theoretically-predicted energy curve. We can obtain the point of interchange by matching Eqs. (1) and (2) without assuming that $\tilde{\delta}$ and $\tilde{h}$ are small: $\tilde{\delta}_c = 2/(\tilde{h}^{-2} - 1)$.

The normalized force-elongation relations are derived as $\tilde{F}_{\text{in}}(\tilde{\delta}) = 2\tilde{h}^{-2}\tilde{\delta}$ and $\tilde{F}_{\text{out}}(\tilde{\delta}) = 2\tilde{\delta} + 2$, respectively, through differentiation of Eqs. (1) and (2) with respect to $\tilde{\delta}$. These forces are plotted in Fig. 2(b) for $\tilde{h} = 0.1$, where the bold curve corresponds to the predicted force curve. This force curve exhibits a discontinuity at the transition point, which results from the difference between the slopes of the two energy curves $\tilde{U}_{\text{in}}(\tilde{\delta})$ and $\tilde{U}_{\text{out}}(\tilde{\delta})$ at the transition point $\tilde{\delta}_c$, as seen in Fig. 2(a). After the transition, the force seems to reach a plateau. In fact, it increases slightly with $\tilde{\delta}$, which is not visible in the plot. This is because the coefficients of the term proportional to $\tilde{\delta}$ in $\tilde{F}_{\text{in}}(\tilde{\delta})$ and $\tilde{F}_{\text{out}}(\tilde{\delta})$ are significantly different. [In Fig. 2(b), the former is 100 ($= \tilde{h}^{-2}$) times as large as the latter.]

We can estimate the ratio γ of the forces before and after transition as $\gamma = \tilde{F}_{\text{out}}(\tilde{\delta}_c)/\tilde{F}_{\text{in}}(\tilde{\delta}_c) = (1 + \tilde{h}^2)/2$. This can be expressed as the following dimensional form:

$$F_{\text{out}}(\delta_c) = (1/2)(1 + (h/d)^2)F_{\text{in}}(\delta_c) \quad (3)$$

Although this expression is derived without assuming that $\tilde{h}$ is small, for practical cases in which $\tilde{h}$ is small, γ is approximately $1/2$. In other words, the force before transition is practically proportional to the force after transition: $F_{\text{out}}(\delta_c) = \gamma F_{\text{in}}(\delta_c)$.

To confirm the validity of Eq. (3), we conducted experiments in the same way as in our previous work.[2] Tensile tests were performed at a constant low pulling speed of 0.5 mm/s for the one-column kirigami sheet [Fig. 1(a)] made of Kent paper with Young's modulus E ($\simeq$ 2.0–2.7) GPa. h, w, and d are of the orders of 0.1, 10, and 1 mm, respectively, while N was set to 10.

We estimated $F_{\text{in}}(\delta_c)$ experimentally as the maximum value F^*_{in} of the experimental force curve in the initial regime [see Fig. 1(b)]. We could not estimate $F_{\text{out}}(\delta_c)$ unambiguously from the experiment because the experimental discontinuous transition was not as sharp as that in the theory. Here, we selected the minimum point F^*_{out} in the experimental curve after the transition as a characteristic value for $F_{\text{out}}(\delta_c)$ [see Fig. 1(b)].

The agreement between theory and experiment is demonstrated in Fig. 3. As expected from Eq. (3), the proportional relation between F^*_{out} and F^*_{in} is confirmed. In fact, this agreement is surprising, considering that the data includes values that do not satisfy the assumption of the simple beam theory, which is required for deriving the energy expressions specified above. The required assumption $w \gg d$ is not well satisfied for the data shown by the open symbols with $(d, w) = (3.5, 15)$, although the other data satisfy $w \geq 5d$.

The value of the slope γ obtained through numerical fitting is $\gamma = 0.586 \pm 0.007$, which is slightly larger than the predicted value. This is because the deviation of the slope from $1/2$ predicted in Eq. (3) is $\tilde{h}^2/2 \simeq 0.0013$–$0.04$ for our experimental parameters. This moderate and apparent discrepancy is reasonable considering the difficulty of obtaining sharp transitions in the experiment, but may have resulted from our estimation of $F_{\text{out}}(\delta_c)$, which tends to yield overestimates: F^*_{out} is expected to be larger than $F_{\text{out}}(\delta_c)$. Note that the elongation δ at $F = F^*_{\text{out}}$ is larger than δ_c [see

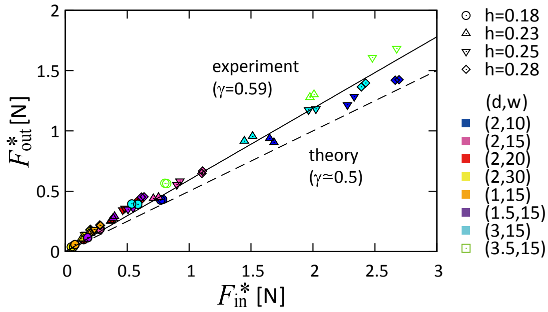

Fig. 3. (Color online) The plot of F^*_{in} vs F^*_{out}. The shape of symbols distinguishes the thickness, while the grayscale (or color) of symbols distinguishes the parameter set (d, w).

Fig. 1(b)], and the force slightly increases after the transition as previously mentioned. (However, this reasoning could be slightly weakened if we recall that the approximate description of the simple beam theory deteriorates when δ becomes comparable to w, as δ at $F = F^*_{\text{out}}$ does in the out-of-plane regime.)

In summary, we have developed our previous theory, deriving an explicit expression for the force discontinuity at the transition [see Eq. (3)]. We confirmed that the expression agrees well with our experimental data. We discussed the discontinuity of the transition observed in our previous and present experiments with paper samples. The discontinuity has also been shown in other experiments,[2,3] while experiments with other materials[4–6] and finite element method calculations[4] have shown continuous transitions. The condition under which the transition becomes continuous or discontinuous is potentially a controversial issue that needs to be addressed in the near future to deepen our understanding of the physics of kirigami, which will be useful for applications of kirigami structures.[7,8] In fact, our theory tacitly assumes the decoupling of the two modes of deformation; the theory without this assumption will be discussed elsewhere.[9]

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